Practical Low Complexity Linear Equalization for MIMO-CDMA Systems

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Abstract—This article first reviews recently proposed techniques for adaptive and direct linear MIMO equalization in the context of MIMO-CDMA systems and in particular with application to a MIMO-extended UMTS-FDD downlink. The focus is thereby mainly on the complexity of the algorithms. The second part of the paper proposes frequency domain (FD) MIMO equalization using the overlap/add FFT method in conjunction with two different low-complexity FD-deconvolution techniques to obtain the equalizer coefficients based on explicit channel impulse response estimates. The effects of imperfect channel estimation are discussed. An architecture for the VLSI implementation of the proposed method is suggested and an estimate of the complexity of the proposed circuit is given in the conclusions.

I. INTRODUCTION

Multiple-Input-Multiple-Output (MIMO) systems [1] have received significant attention in recent years as a means to increase the spectral efficiency of communication systems. Originally the concept has mostly been applied to narrow-band systems, however in the past two years it has made its way into many broad-band systems. MIMO is now being considered for 3G mobile communication and specifically as an extension to UMTS [2] and to HSDPA. However, it is also known that in practice the performance of such systems is heavily limited by self- and multiple-access interference through multipath propagation, which on the other hand provides additional diversity and increases the richness of the channel. Unfortunately with basic low-complexity receivers (such as a space-time RAKE) the adverse effect of the interference is in general the dominant factor [3], [4].

While joint maximum likelihood detection would be the optimal solution, it is not feasible in practice due to its exponential complexity. Besides numerous multi-user-detection schemes *linear chip-spaced equalization* is currently being considered by the industry as a practical approach to improve the performance of the standard single-input-single-output (SISO) UMTS-FDD downlink under multipath propagation conditions (e.g.: [16]). Linear MIMO equalization is used to achieve higher data rates on the downlink through spatial multiplexing with increased resilience against interference [7], [6]. However, despite quite

promising performance results the complexity of such a system is still considerably high and the various equalization techniques differ significantly in the number of operations they require to compute the equalizer coefficients and to apply them to the received signal.

This paper first gives a brief overview of candidate algorithms for linear MIMO-equalization in general. The focus is thereby on the analysis of the computational complexity of the algorithms and their suitability for VLSI implementation. The third section briefly describes the MIMO-extended UMTS-FDD downlink system which we consider in this paper. In section four of the paper two low-complexity equalization methods based on deconvolution of an estimated channel are described and discussed in the context of a MIMO-CDMA system. The effects of imperfect channel estimation are also discussed. The fifth section describes a high-level VLSI architecture for their realization before the paper is concluded.

II. COMPLEXITY OF LINEAR MIMO EQUALIZATION

Linear equalization of MIMO channels is an extensively studied subject that received some attention in other fields even before the introduction of MIMO concepts in wireless communication (see references in [5]). Numerous authors have derived and analyzed various linear equalization techniques, often based on concepts and ideas that are well known from the SISO case. Almost all proposed algorithms can be categorized based on their learning strategy, which use an *estimate of the channels impulse response* or *direct adaption* to a transmitted pilot sequence (blind methods are not considered here). In addition to that it is noted that linear convolution can be carried out in the *time domain (TD)* or in the *frequency domain*, which is known from the audio signal processing field to greatly reduce the implementation complexity.

The complexities of various algorithms are presented in table I. It is distinguished between the number of operations for the application of the equalizer to the received signal and the number of operations that are required for the adaptation/computation of the equalizer coefficients. The parameters are the number of

transmit antennas N_{Tx} and receive antennas N_{Rx} as well as the length of the equalizer L_{eq} and in some cases the adaptation interval G in samples. The number of multiplications is used as a measure, while it is noted that in some cases (e.g. RLS) other operations, such as CORDICs are typically used. However in general the number of operations remains at least similar. In the fourth column an example is given which is relevant to the application that is introduced in the next section where a MIMO-CDMA system with four transmit and four receive antennas is considered with a 32 tap chip-spaced equalizer.

From the table it is noted that direct TD-deconvolution is not feasible even if the special block toeplitz structure of the channel matrix is taken into account. The RLS based adaptive approach offers some reduction in complexity and maintains good performance, however it is still far from suitable for VLSI implementation and suffers from numerical problems. Currently, time domain adaptive LMS methods are the only practical approach. In fact it is noted that a similar system configuration has been considered in [6], [7], suggesting the TD-NLMS and the therein newly derived TD-NLMS-G method to achieve low complexity.

An alternative approach to the equalization based on a linear convolution in the time domain is the equalization in the frequency domain. It is known from the audio signal processing field that the application of the equalizer coefficients to the signal (convolution) can be performed very efficiently in the frequency domain (using the overlap/add or overlap/save method) without any performance loss. It can be shown that the savings are even higher in the MIMO case and that the FD-convolution becomes efficient even for very short equalizer lengths. For the adaptation of the coefficients, FD methods can also be used to further simplify the well known LMS algorithms. An RLS algorithm based on the frequency domain representation of the MIMO channel has also been suggested. The corresponding references are given in table I.

III. LINEAR EQUALIZATION IN MIMO-UMTS

A. System Overview and Model

The system under consideration is based on the UMTS-FDD downlink. A schematic overview of the transmitter is shown on the left side of figure 1. The data signal for each of the K active users is $QPSK$ modulated before being demultiplexed into N_{Tx} parallel streams. The spatial rate is therefore equal to the number of antennas. Each of these streams is then upsampled to the chiprate of $F_C = 3.84$ Mchips/s and multiplied with a user-specific Orthogonal Variable Spreading Factor (OVSF) channelization code. The length of the code determines the spreading factor (SF) and is derived from the desired data rate B as $SF = \frac{2 \cdot F_C \cdot N_{Tx}}{B}$. Subsequently the corresponding streams of all users¹ are summed and a separate pilot is added for each of the

transmit antennas. The resulting signals are spread with a cell-specific primary scrambling code (PSC) to form the set of N_{Tx} transmitted signal vectors $\mathbf{x}_q[t] = [x_q[t] \ x_q[t-1] \ \dots]^T$. A more detailed description is given in [2].

The frequency selective MIMO channel (with length L_c) is described for each spatial path from antenna $q = 1, \dots, N_{Tx}$ to antenna $p = 1, \dots, N_{Rx}$ separately through a vector $\mathbf{h}_{p,q} = [h_{p,q}[1] \ h_{p,q}[2] \ \dots \ h_{p,q}[L_c]]^T$ whose entries are assumed to be uncorrelated, zero mean Gaussian distributed with their respective variances defined through a single power delay profile across all spatial subchannels². A set of L_{eq} received samples is now described by

$$\mathbf{y} = \mathbf{Q}\mathbf{x} + \mathbf{n}.$$

The vectors $\mathbf{x}$ and $\mathbf{y}$ are thereby formed by stacking the vectors $\mathbf{x}_q$ and $\mathbf{y}_p$ that contain the transmitted and received samples at the antennas q and p respectively. The matrix $\mathbf{Q}$ is obtained by first arranging the vectors $\mathbf{h}_{p,q}$ into toeplitz matrices $\mathcal{H}_{p,q}$ which are then stacked to fully describe the frequency selective MIMO channel.

$$\mathcal{H}_{p,q} = \begin{bmatrix} \mathbf{h}_{p,q}^T & 0 & 0 & \dots & 0 \\ 0 & \mathbf{h}_{p,q}^T & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & \mathbf{h}_{p,q}^T & 0 \\ 0 & \dots & 0 & 0 & \mathbf{h}_{p,q}^T \end{bmatrix} \quad (1)$$

$$\mathbf{Q} = \begin{bmatrix} \mathcal{H}_{1,1} & \dots & \mathcal{H}_{1,N_{Tx}} \\ \mathcal{H}_{2,1} & \dots & \mathcal{H}_{2,N_{Tx}} \\ \vdots & \ddots & \vdots \\ \mathcal{H}_{N_{Rx},1} & \dots & \mathcal{H}_{N_{Rx},N_{Tx}} \end{bmatrix} \quad (2)$$

A block diagram of the receiver is shown on the right side of figure 1. Linear chip-spaced equalization is first applied to reduce the effect of multipath in the channel to restore the orthogonality of the channelization OVSF codes. At the same time the equalizer performs linear spatial separation at the chip-level of the transmitted signals based on an MMSE or ZF criterion. For reference the corresponding TD-solution for the q th MISO MMSE equalizer is computed as:

$$\mathbf{c}_q^{TD} = \left(\mathbf{Q}\mathbf{Q}^H + \frac{N_{Tx}}{SNR}\mathbf{I} \right)^{-1} \mathbf{Q} \mathbf{e}_{(L_{eq}+L_c-1)(q-1)+L_{eq}/2}} \quad (3)$$

The vectors $\mathbf{e}_{(L_{eq}+L_c-1)(q-1)+D}$ thereby consist of N_{tx} stacked vectors of length $L_{eq}+L_c-1$ which describe the desired impulse response from each of the transmit antennas to the output of the q th MISO equalizer. Subsequently the signal is despread. To reduce the effect of residual spatial interference and to allow for spatial noise whitening at the symbol level (not employed in this work) a spatial-only maximum likelihood detector is used for detection.

¹Note that not all users need to have the same spatial rate.

²For relatively small antenna spacing this is a reasonable assumption.

Algorithm	Equalizer Application	Equalizer Adaptation	Example: 4×4 $L_{eq} = 32$	Adaptation Interval
TD-Deconv. [8]	$N_{Tx} N_{Rx} L_{eq}$	$> \frac{3(N_{Rx} N_{Tx}^2 L_{eq}^3)}{G}$	Appl.:512 Adapt.:25'000	$G = 256$
TD-RLS [9]	$N_{Tx} N_{Rx} L_{eq}$	$\frac{N_{Rx} L_{eq} (\frac{5}{2} + N_{Tx})}{2G} \frac{1}{G}$	Appl.:512 Adapt.:800	$G = 32$
TD-(N)LMS [6], [7], [9]	$N_{Tx} N_{Rx} L_{eq}$	$N_{Tx} N_{Rx} L_{eq} + N_{Rx}$	Appl.:512 Adapt.:516	$G = 1$
TD-NLMS-G [6], [7]	$N_{Tx} N_{Rx} L_{eq}$	$\frac{N_{Tx} N_{Rx} L_{eq}}{G} + N_{Rx}$	Appl.:512 Adapt.:36	$G = 16$
FD-RLS [10], [11]	$(N_{Tx} + N_{Rx}) \log_2(2L_{eq}) + 2N_{Tx} N_{Rx}$	$3N_{Rx} (N_{Rx} + 1 + \frac{3}{2} N_{Tx}) + \log_2(2L_{eq}) N_{Tx}$	Appl.:80 Adapt.:138	$G = 1$
FD-FLMS (SISO: [13])	$(N_{Tx} + N_{Rx}) \log_2(2L_{eq}) + 2N_{Tx} N_{Rx}$	$(N_{Tx} + 2N_{Tx} N_{Rx}) \log_2(2L_{eq}) + 2N_{Tx} N_{Rx}$	Appl.:80 Adapt.:248	$G = 1$
FD-UFLMS (SISO: [14])	$(N_{Tx} + N_{Rx}) \log_2(2L_{eq}) + 2N_{Tx} N_{Rx}$	$N_{Tx} \log_2(2L_{eq}) + 2N_{Tx} N_{Rx}$	Appl.:80 Adapt.:56	$G = 1$
FD-Direct cyclic	$(N_{Tx} + N_{Rx}) \log_2(2L_{eq}) + 2N_{Tx} N_{Rx}$	$(3N_{Tx}^2 + 5N_{Tx}) \frac{N_{Rx} L_{eq}}{G} + \frac{N_{Tx} N_{Rx} L_{eq}}{2G} \log_2(L_{eq})$	Appl.:80 Adapt.:37	$G = 256$
FD-Direct approx. window [12]	$(N_{Tx} + N_{Rx}) \log_2(2L_{eq}) + 2N_{Tx} N_{Rx}$	$(3N_{Tx}^2 + 5N_{Tx}) \frac{2N_{Rx} L_{eq}}{G} + \frac{N_{Tx} N_{Rx} L_{eq}}{G} \log_2(2L_{eq})$	Appl.:80 Adapt.:80	$G = 256$

TABLE I
COMPLEXITY COMPARISON IN COMPLEX MULTIPLICATIONS PER SAMPLE

Fig. 1. System Overview

IV. DIRECT FREQ. DOMAIN METHODS

A. Frequency Domain Convolution

To reduce the complexity of the linear convolution in the equalizer it can be carried out in the frequency domain using the overlap/add or overlap/save method. In this paper we will use the first approach. Extending it from the SISO case to the multichannel case is straight forward. At each receive antenna a set of L_{eq} samples is collected and zero extended to a length of $2L_{eq}$. The blocks are independently transformed into frequency domain. If $2L_{eq}$ is chosen as a power of four, an efficient radix-4 FFT can be used. Furthermore, the fact that half of the input consists of zeros can be used to prune some branches in the FFT. Similar to the OFDM case the oversampled equalizer coefficients are applied to each frequency component separately, requiring $2L_{eq}$ independent Matrix/Vector multiplications. The

result is transformed back into time domain. The first L_{eq} samples are added to the last L_{eq} samples of the previous block and forwarded to the despreaders. The second block is buffered for the overlap/add operation with the next block. The fact that an $N_{Tx} \times N_{Rx}$ matrix equalizer requires only N_{Rx} FFTs and N_{Tx} IFFT leads to the fact that the MIMO case allows for proportionally even higher savings in terms of operations per sample than the SISO case.

B. Windowed Freq. Domain Deconvolution

The windowed frequency domain deconvolution is based on the simple observation that in the SISO case the infinite length MMSE equalizer can always be obtained in the frequency domain as

$$H(f) = \mathfrak{F}\{\mathbf{h}\}, \quad T(f) = \mathfrak{F}\{\delta[t - D]\} \quad (4)$$

$$C(f) = \frac{T(f)H^*(f)}{\left(\frac{1}{SNR} + H(f)H^*(f)\right)}, \quad \mathbf{c} = \mathfrak{F}^{-1}\{C(f)\} \quad (5)$$

whereby $\delta[t - D]$ is the target impulse response of the combined channel/equalizer system and the infinite length vector $\mathbf{c}$ contains the equalizer coefficients. The operators $\mathfrak{F}\{\bullet\}$ and $\mathfrak{F}^{-1}\{\bullet\}$ denote the Fourier transform and its inverse.

In order to obtain a finite length equalizer solution as an approximation of equations 4 and 5 the channel impulse response is first zero extended to a length of $2L_{eq}$: $\hat{\mathbf{h}} = [\mathbf{h}^T \quad \mathbf{0}_{1 \times 2L_{eq} - L_c}]^T$. The length L_{eq} equalizer impulse response is described as its own periodic repetition which is then componentwise multiplied ($\odot$) with a square window $\mathbf{w}$: $\tilde{\mathbf{c}} = \tilde{\mathbf{c}} \odot \mathbf{w}$ with

$$\tilde{\mathbf{c}} = \begin{bmatrix} \mathbf{c} \\ \mathbf{c} \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} \mathbf{1}_{L_{eq} \times 1} \\ \mathbf{0}_{L_{eq} \times 1} \end{bmatrix} \quad (6)$$

The exact windowed frequency domain solution for the equalizer coefficients $\mathbf{c}$ can now be formulated based on the DFT:

$$\left(\underbrace{\text{DFT}\{\tilde{\mathbf{c}}\}}_{\tilde{\mathbf{C}}[f]} \star \underbrace{\text{DFT}\{\mathbf{w}\}}_{\mathbf{W}[f]} \right) \underbrace{\text{DFT}\{\hat{\mathbf{h}}\}}_{\hat{\mathbf{H}}[f]} = \underbrace{\text{DFT}\{\delta[t - D]\}}_{\hat{\mathbf{T}}[f]} \quad (7)$$

The problem is that, as a consequence of the cyclic convolution with the frequency domain response of the square window, expression 7 consists of a set of coupled equations. This prevents a simple solution to the problem.

The goal is now to find a more suitable window $\tilde{\mathbf{w}}$ that leads to a more convenient (preferably decoupled) set of equations in 7, while still representing a good approximation to the original rectangular window. An appropriate choice, which also leads to a rather simple hardware implementation is given in the frequency domain as

$$\tilde{\mathbf{W}}[f] = \text{DFT}\{\tilde{\mathbf{w}}\} = \begin{cases} 1 & , f = 0 \\ \exp\left(-j\frac{2\pi}{4}\right) & , f = 1 \\ 0 & , 1 < f < 2L_e \end{cases} \quad (8)$$

A set of L_{eq} mutually decoupled pairs of equations is obtained to which an independent MMSE solution can be found as

$$\mathbf{C}_{MMSE}[f] = \left(\frac{1}{SNR} + \tilde{\mathbf{H}}^H[f] \tilde{\mathbf{H}}[f] \right)^{-1} \tilde{\mathbf{H}}^H[f] \tilde{\mathbf{T}}[f] \quad (9)$$

with vectors $\tilde{\mathbf{H}}$ and $\tilde{\mathbf{T}}$ defined as

$$\begin{aligned} \tilde{\mathbf{H}}[f] &= [\tilde{\mathbf{W}}[0] \hat{\mathbf{H}}[2f] \quad \tilde{\mathbf{W}}[1] \hat{\mathbf{H}}[2f+1]]^T \\ \tilde{\mathbf{T}}[f] &= [\hat{\mathbf{T}}[2f] \quad \hat{\mathbf{T}}[2f+1]]^T \end{aligned}$$

For the MIMO case the matrix $\tilde{\mathbf{H}}$ replaces $\tilde{H}$ in equation 9. It is constructed by transforming the zero extended impulse responses from each transmit to each receive antenna $\hat{\mathbf{h}}_{p,q}$ separately into frequency domain using the FFT to obtain $\hat{\mathbf{H}}_{p,q}[f]$. These are then stacked similarly to equation 2 to obtain the matrices $\hat{\mathbf{H}}[f]$ that describe the MIMO channel for each frequency component³. With a similar extension to $\tilde{\mathbf{T}}$ a matrix $\mathbf{C}_{MMSE}[f]$ is obtained that represents the MIMO equalizer solution at each frequency component

$$\begin{aligned} \tilde{\mathbf{H}}[f] &= [\tilde{\mathbf{W}}[0] \hat{\mathbf{H}}[2f] \quad \tilde{\mathbf{W}}[1] \hat{\mathbf{H}}[2f+1]]^T \\ \tilde{\mathbf{T}}[f] &= [\hat{\mathbf{T}}[2f] \mathbf{I}_{N_{Tx} \times N_{Tx}} \quad \hat{\mathbf{T}}[2f+1] \mathbf{I}_{N_{Tx} \times N_{Tx}}]^T \\ \mathbf{C}_{MMSE}[f] &= \left(\frac{N_{Tx}}{SNR} \mathbf{I} + \tilde{\mathbf{H}}^H[f] \tilde{\mathbf{H}}[f] \right)^{-1} \tilde{\mathbf{H}}^H[f] \tilde{\mathbf{T}}[f] \end{aligned} \quad (10)$$

The result of the above equation can either be transformed back into time domain to directly obtain the L_{eq} equalizer coefficients or it can be used together with the overlap/add frequency domain equalization. However, in the latter case a $2 \times$ upsampling of the coefficients has to be performed either directly in the frequency domain through a convolution with an appropriate window or through a transformation into time domain, followed by a windowing operation and a transformation back into frequency domain. As opposed to the equalizer itself, the first alternative is generally more efficient to implement. This is due to the fact that the coefficients of the convolution are constant and that the operation can be approximated quite accurately with a relatively short FIR filter.

Simulation results that show the performance of the frequency domain approximation are given in figure 2. The power/delay profile of the channel was chosen as a chip-spaced version of the ‘‘PedestrianB’’ channel with Rayleigh fading coefficients. The system uses four transmit and four receive antennas and a spreading factor of 32 to provide a raw data rate of 1Mbps to each of the $K = 1, 8$ and 16 concurrent users. The E_b/N_o is independent of the number of users, which goes along with an increase in SNR as more users become active. With only a single active user the FD-equalizer follows closely the full TD-solution and no error floor is visible. However when 8 or 16 users are concurrently active the performance of the FD-solution starts to degrade and the residual interference shows up as an error floor around $2 \cdot 10^{-3}$ and 10^{-4} respectively.

In these simulations perfect channel knowledge at the receiver was assumed. However in practice channel estimation errors in a simple correlation based channel estimator will result from three sources:

- 1) Thermal noise
- 2) Self and multiple-access interference between the pilots and the data channels
- 3) Aging of the estimated channel profile depending on the Doppler spread

³Under the assumption of a cyclic convolution channel

Fig. 2. BER v.s. E_b/N_0 for TD-MMSE and the approximate windowed FD-MMSE equalizer

To a certain extent the first two problems can be reduced by averaging over a longer period and by increasing the pilot-power to user-power ratio. The second measure will of course result in additional interference to the user channels which however can be mitigated at the receiver as the pilot signals are known [15]. For the simulations in figure 3 the overly optimistic assumption is made that the Rayleigh fading channel is constant throughout the duration of a burst and is perfectly known to the receiver at a time instant t . The computation of the equalizer coefficients requires the time Δt during which the channel changes. The correlation coefficient between the new and the old channel is known to be $\alpha = J_0(2\pi F_d \Delta t)$. Results are plotted for $\alpha = 1, 0.999, 0.996, 0.991$, which roughly correspond to a computation delay of $\Delta t = 256$ chips at Doppler spreads of $F_d = 0, 150, 300, 450$ Hz. While under these idealistic conditions good performance is achieved even for large Doppler spreads it should be noted that the results also indicate that very accurate channel estimation is required to obtain good results, which in practice is the main drawback of the algorithm.

For an assessment of the sensitivity of the algorithm to channel estimation noise it is modeled as additive uncorrelated zero-mean Gaussian noise with variance σ^2 . At the same time the correlation coefficient between the actual and the estimated channel is assumed to be $\alpha = 0.999$. The results are shown in figure 4. It is observed that the performance of the system degrades sharply with an increase in the channel estimation error. At least 17dB signal to noise ratio in the channel estimate are necessary to

Fig. 3. Influence of Doppler spread

achieve a significantly better performance than the RAKE with perfect channel knowledge. It is also noted that as the SNR increases the error floor of the equalizer eventually also increases. This is explained by the fact that the channel mismatch can be partially accounted for in the equalizer computation by modeling the resulting distortion as an increase in AWGN, leading to a decreased effective SNR in equation 10. If this is not explicitly modeled the MMSE solution is known to approach the ZF solution in the high SNR region and the equalizer causes an enhancement of the mismatch noise.

C. Cyclic Frequency Domain Solution

The *cyclic FD-equalizer* is a special case of the approximate windowed solution where a flat window is chosen as $\tilde{\mathbf{w}} = \mathbf{1}_{2L_{eq} \times 1}$. This is essentially an approximation to 5 by simply assuming that the convolution in the channel can be modeled as a cyclic convolution of the transmitted signal with a zero extended channel impulse responses $\tilde{\mathbf{h}}_{p,q} = [\mathbf{h}_{p,q}^T \quad \mathbf{0}_{1 \times L_{eq} - L_c}]^T$ of length L_{eq} . In the absence of a cyclic prefix (which is assumed here) this is of course only an approximation and the accuracy of this assumption naturally depends on the length of the zero extension, i.e. the relation between the length of the channel and the length of the equalizer. However, equations 4 and 5 now become even simpler :

$$\mathbf{C}_{MMSE}[f] = \hat{T}[2f] \left(\frac{N_{Tx}}{SNR} \mathbf{I} + \tilde{\mathbf{H}}^H[f] \tilde{\mathbf{H}}[f] \right)^{-1} \tilde{\mathbf{H}}^H[f]. \quad (11)$$

Fig. 4. Performance of imperfect channel estimation

Fig. 5. Cyclic FD-equalizer assuming perfect channel knowledge

In addition to that, only FFTs of the length of the equalizer are required to obtain the frequency domain representations of $\mathbf{h}_{p,q}$.

Simulations assuming perfect channel knowledge are shown in figure 5. For comparison the approximate windowed and the cyclic solution are given side by side. Similar to the case where the equalizer solution was affected by channel estimation mismatch, an increase in the BER is noted in the high SNR region with a similar explanation. However, in this case the unaccounted noise originates from the assumption of a circular convolution in the channel. This interference can be estimated and can be taken into account in equation 11, to avoid this kind of approximation-noise enhancement. It should also be noted that the performance of the cyclic approximation degrades significantly as the length of the channel approaches the length of the equalizer. In this situation the slightly more complex *approximate window FD-solution* can deliver better performance and should be preferred.

V. HARDWARE

A. Overview

A block diagram of the high-level architecture of an equalizer is shown in figure 6. It consists of five main subcircuits:

- 1) The Channel Profile Estimation unit computes the channel impulse response by correlating the received signals from each receive antenna with the four known pilot symbols. Due to the simplicity of the training sequences this procedure can be realized very efficiently in hardware.

Fig. 6. High-level Architecture

- 2) A radix-4 FFT/IFFT block with a block length of $2L_{eq}$ and the ability to perform matrix $\times$ vector multiplications. It performs the equalization of the received signal and is also used to transform the $N_{Tx}N_{Rx}$ channel impulse responses into frequency domain for the FD-deconvolution.
- 3) An overlap/add block which mainly consists of $2L_{eq}N_{Tx}$ words of memory to carry out the overlap/add operation and to handle the parallel to serial conversion of the result.
- 4) A matrix inversion unit to compute equation 10. For a reasonably small number of antennas (2-6) this operation can be realized efficiently using for example the Riccati [17] recursion.
- 5) An upsampling unit that generates the upsampled frequency domain representation of the length L_{eq} MIMO equalizer for the overlap/add FFT method